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COHERENT-TRAPPING STATES OF THREE-LEVEL TWO-MODE SYSTEMS WITH MULTIPHOTON TRANSITIONS

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The coherent-trapping states of three-level two-mode systems with multiphoton transitions have been found. All three types of atomic transition configurations have been considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Состояния когерентного пленения населенностей уровней в многофотонных трехуровневых двухмодовых системах

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Найдены состояния когерентного пленения населенностей уровней в многофотонных трехуровневых двухмодовых системах. Исследованы все типы конфигураций переходов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

In recent years, the study of coherent population trapping has received a great deal of interest /1-14/. This phenomenon consists in impossibility to excite an atom to a given level in spite of the existence of both a radiation field and allowed transitions from other levels initially occupied. Coherent population trapping is explained to occur due to the destructive interference between two or more transition channels. A pure two-level system cannot exhibit trapping. Coherent population trapping in a three-level atomic system has been noted under the two-photon resonance condition and has been well studied semiclassically for a lambda-type atomic system with one-photon transitions /1-7/. It has been shown that one of the levels involved can even be a member of a continuum, and coherent trapping remains possible /8-10/. The conditions under which coherent population trapping in two-photon ionization occurs have been obtained /11-12/. The possibility of coherent trapping with the quantized cavity field has been examined for all the possible types of single-photon-transition three-level atoms /13-14/ by using the dressed state formalism.

In this paper we present a treatment of coherent population trapping with the quantized cavity field. We find the so-called coherent trapping states of a multiphoton-transition three-level two-mode system using a procedure different from the Yoo-Eberly treatment for the single-photon-transition case [14] and the treatment for the multiphoton lambda-configuration system [15].

Our system in its most general form consists of a three-level atom and a quantized electromagnetic field of two excited cavity modes. We treat this system as closed, i.e., no coupling of the atom with the radiation field modes of free space. The atom and the excited cavity modes couple to each other by an effective interaction with allowed multiphoton transitions of atomic levels 1↔3 and 2↔3 but not of 1↔2. The rotating wave approximation is used.

There are three distinct atomic level configurations in which level 3 is intermediate, upper or lower. We call these configurations cascade-, lambda-, and V-types, respectively, and treat them formally in the same manner.

The system is described by the Hamiltonian

$$H = H_A + H_F + H_{AF} , \tag{1}$$

where the free atomic part H_A and the free field part H_F are

$$H_A = \sum_{j=1}^{3} \hbar\Omega_j R_{jj} \tag{2}$$

and

$$H_F = \sum_{\alpha=1}^{2} \hbar\omega_\alpha a_\alpha^+ a_\alpha . \tag{3}$$

The operator R_{jj} describes the population of atomic level j , a_α^+ and a_α are the creation and annihilation operators of a photon in mode α, Ω_j is the jth atomic level frequency, while ω_α is the frequency of mode α . The interaction part H_{AF} of the Hamiltonian H is given, depending on the three possible types of the atomic level configurations, by

$$H_{AF} = \hbar g_1 a_1^{m_1} R_{31} + \hbar g_2 a_2^{m_2} R_{23} + \text{h.c.} \qquad \text{(cascade type)}, \tag{4.1}$$

$$H_{AF} = \hbar g_1 a_1^{m_1} R_{31} + \hbar g_2 a_2^{m_2} R_{32} + \text{h.c.} \qquad \text{(lambda type)}, \tag{4.2}$$

$$H_{AF} = \hbar g_1 a_1^{m_1} R_{13} + \hbar g_2 a_2^{m_2} R_{23} + \text{h.c.} \qquad \text{(V-type)}, \tag{4.3}$$

where h.c. is hermitian conjugate, g_1 and g_2 are coupling constants, m_1 and m_2 are the photon multiples of the atomic transitions, and R_{ij} describes the atomic transition from level j to level i $(i \neq j)$. The operators R_{ij} $(i, j = 1, 2, 3)$ are defined by

$$R_{ij} = |i\rangle_A \, {}_A\langle j| . \qquad (5)$$

They obey the relation

$$R_{ij} \, |k\rangle_A = \delta_{kj} \, |i\rangle_A . \qquad (6)$$

Here $|i\rangle_A$ $(i = 1,2,3)$ is the state vector of atomic level i. We impose the two-mode multiphoton resonance condition, where the sum (cascade type) or difference (lambda and V-types) of two mode-frequency multiples is exactly on resonance with the atomic frequency difference between levels 1 and 2

$$\Omega_2 - \Omega_1 = m_1\omega_1 + m_2\,\omega_2 \qquad \text{(cascade type)}, \qquad (7.1)$$

$$\Omega_2 - \Omega_1 = m_1\omega_1 - m_2\omega_2 \qquad \text{(lambda type)}, \qquad (7.2)$$

$$\Omega_2 - \Omega_1 = m_2\omega_2 - m_1\omega_1 \qquad \text{(V-type)} \qquad (7.3)$$

Now we call a state of the system, $|\phi\rangle$, a coherent-trapping state if $|\phi\rangle$ satisfies the following conditions [14]:

$$\text{i)} \quad |\phi\rangle = |\phi_A\rangle \otimes |\phi_{F_1}\rangle \otimes |\phi_{F_2}\rangle , \qquad (8)$$

and

$$\text{ii)} \quad \begin{aligned} |\phi(t)\rangle &\equiv \exp(-iHt/\hbar)\,|\phi\rangle = \\ &= \exp(-iH_A t/\hbar)\,|\phi_A\rangle \otimes \exp(-iH_F t/\hbar)\,|\phi_F\rangle , \end{aligned} \qquad (9)$$

where $|\phi_A\rangle$ is an atomic state made by a proper linear superposition of levels 1 and 2, and $|\phi_F\rangle \equiv |\phi_{F_1}, \phi_{F_2}\rangle$ is a field state different from the states consisting of only the number states $|n_1, n_2\rangle$ with $n_1 < m_1$ or $n_2 < m_2$. Taking into account the resonance condition described by eqs. (7.1)-(7.3) we can easily show that the coherent-trapping condition (ii) is equivalent to

$$H_{AF}\,|\phi\rangle = 0 . \qquad (10)$$

Because level 3 is not involved with $|\phi_A\rangle$ we can write $|\phi_A\rangle$ as

$$|\phi_A\rangle = u_1|1\rangle_A - u_2|2\rangle_A, \quad |u_1|^2 + |u_2|^2 = 1. \tag{11}$$

Using eqs. (4.1)-(4.3), (8) and (11) we get from eq. (10)

$$g_1 u_1 a_1^{m_1}|\phi_F\rangle = g_2 u_2 a_2^{+m_2}|\phi_F\rangle \qquad \text{(cascade type)}, \tag{12.1}$$

$$g_1 u_1 a_1^{m_1}|\phi_F\rangle = g_2 u_2 a_2^{m_2}|\phi_F\rangle \qquad \text{(lambda type)}, \tag{12.2}$$

$$g_1 u_1 a_1^{+m_1}|\phi_F\rangle = g_2 u_2 a_2^{+m_2}|\phi_F\rangle \qquad \text{(V-type)}. \tag{12.3}$$

Equations (12.1)-(12.3) determine $|\phi_F\rangle \equiv |\phi_{F_1}, \phi_{F_2}\rangle$, the values of u_1 and u_2 and thus $|\phi_A\rangle$ for all the three types of atomic configurations. It is easy to solve these equations. As a result we obtain for the lambda-type system the field coherent-trapping states

$$|\phi_F\rangle = |z_1\rangle_{coh} \otimes |z_2\rangle_{coh} \quad . \tag{13}$$

Here $|z_1\rangle_{coh}$ and $|z_2\rangle_{coh}$ are Glauber coherent states of modes 1 and 2, respectively. The amplitudes u_1 and u_2 of the corresponding atomic coherent-trapping states should be

$$u_1 = \frac{g_2 z_2^{m_2}}{\sqrt{|g_1 z_1^{m_1}|^2 + |g_2 z_2^{m_2}|^2}}, \quad u_2 = \frac{g_1 z_1^{m_1}}{\sqrt{|g_1 z_1^{m_1}|^2 + |g_2 z_2^{m_2}|^2}}. \tag{14}$$

For the cascade- and V-type systems we find no coherent-trapping state.

Thus, in this paper we have investigated the coherent trapping effect in the multiphoton three-level two-mode systems. All three types of atomic transitions have been considered. It has been shown that coherent-trapping states exist only for the lambda-type system. Our results have been straight-forwardly obtained from the interaction Hamiltonian without using the dressed state formalism.

REFERENCES

1. *Alzetta G., Gozzini A., Moi L., Orriols G. – Nuovo Cim., 1976, B36, p.5.*
2. *Orriols G. – Nuovo Cim.,1979, B53, p.1.*
3. *Gray H.R., Whitley R.M., Stroud C.R. Jr. – Opt. Lett., 1978, 3, p.218.*
4. *Radmore P.M., Knight P.L. – J. Phys., 1982, B15, p.561.*
5. *Dalton B.J., Knight P.L. – Opt. Commun., 1982, 42, p.411.*

6. *Swain S. – J. Phys., 1982, B15, p.3405.*
7. *Minogin V.G., Rozdestvensky Yu. V. – Sov. JETP, 1985, 88, p.1950.*
8. *Rzazewski K.. Eberly J.H. – Phys. Rev. Lett., 1981, 47, p.408.*
9. *Rzazewski K., Eberly J.H. – Phys. Rev., 1983, A27, p.2026.*
10. *Knight P.L., Lauder M.A., Radmore P.M., Dalton B.J. – Acta Physica Austriaca, 1984, 56, p.103.*
11. *Radmore P.M., Knight P.L. – Phys. Lett., 1984, A102, p.180.*
12. *Deng Z. – Phys. Lett., 1984, A105, p.43.*
13. *Yoo H.I. Coherence and Quantum Optics 5: proc. 5 Rochester Conf., June 13-15,1983 , New York; London, 1984.*
14. *Yoo H.I., Eberly J.H. – Phys. Rep., 1985, 118, p.239.*
15. *Aliskenderov E.I., Fam Le Kien, Shumovsky A.S. – Phys. Lett., 1987, A120, p.401.*

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